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Note

Exact Two-Particle S-Matrix of Quantum Sine-Gordon Solitons

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Abstract. The exact and explicit formulas for the quantum S-matrix elements of the soliton-antisoliton scattering which satisfy unitarity and crossing conditions and have correct analytical properties are constructed. This S-matrix is in agreement with the massive Thirring model perturbation theory and with the semiclassical sine-Gordon results.

As it is known the sine-Gordon model, i.e. the model of the field $\phi(x)$ in $(1+1)$ space-time, described by Lagrangian density

$$L=\frac{1}{2}(\partial\mu\phi)^2+\frac{m_0^2}{\beta^2}\mathrm{Cos}(\beta\phi) \tag{1}$$

has an infinite number of conservation lows both on the classical and on the quantum levels [1, 2]. This gives hard restrictions on particle scattering processes in this model. Namely, the set of particles constituting the final state of scattering and the set of their momenta coincide with the sets of the particles and momenta of the initial state, i.e. the particles may only exchange their momenta in the process of collision [2, 3, 9, 10].

The particle spectrum in model (1) consists of a soliton, an antisoliton (this particles we shall denote as A and $\bar{A}$) and a certain number of soliton-antisoliton bound states. The masses of the latter are described by the formula:

$$m_k=2m\,\mathrm{Sin}\frac{k\gamma}{16}; \qquad k=1,2,\ldots<\frac{8\pi}{\gamma}, \tag{2}$$

where m is a soliton mass and $\gamma=\beta^2\left[1-\frac{\beta^2}{8\pi}\right]^{-1}$. This expression was obtained by semiclassical quantization of double-soliton solution of the classical sine-Gordon equation but seems to be exact [5, 3]. In the following consideration we shall always treat (2) as exact values of particle masses. Let us consider two-particle soliton-antisoliton scattering. Corresponding S-matrix element consists of two components $S_1(s)$ and $S_2(s)$ only [$s=(p_1+p_2)^2$ where p_1 and p_2 are the momenta of initial particles], describing two possible direct channels of reaction: the forward scattering (penetration) and the backward scattering (reflection) respectively. $S_1(s)$ and $S_2(s)$