Commun. math. Phys. 55, 179—182 (1977)

Communications in Mathematical Physics
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Some Global Properties of Massless Free Fields

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Abstract. Elementary group-theoretical considerations show that global solutions to the massless free field equations are functions on the bundle of twistor dyads, rather than the bundle of conformal spin frames. Only in certain degenerate cases may they be thought of as ordinary spinor fields. This is the origin of the "Grgin discontinuity".

I. Introduction

It has been known for some time that global solutions to the massless free field equations

$$\nabla^{AA'}\varphi_{A'B'\ldots C'}=0 \tag{1}$$

cannot generally be constructed on the conformal compactification of Minkowski space: if one attempts to extend a local solution to cover any closed null geodesic, a point is encountered at which the one-sided limits differ by a factor $i^{-2|s|-2}$, where s is the helicity of the field. This phenomenon was first noticed by Grgin [1], and has recently been discussed by one of us [2] from the point of view of U-spinors. In this note, we show that the "Grgin discontinuity" arises from purely group-theoretical considerations—in particular from the fact that ψ is required to be a density of conformal weight $-|s|-1$ in order that the field equations be invariant under SU(2, 2). Given the correct transformation properties under the conformal group, the discontinuity will occur whether or not (1) is satisfied. Although global solutions to the field equations *do* exist for all values of s [3], they cannot consistently be thought of as spinor fields in the usual sense.

This all turns out to be intimately related to the question of when a twistor dyad can be identified with a spin-frame, and this is the point of view from which we shall approach the question.

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** Supported in part by the University of Kansas Research Fund and by the Science Research Council