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Large-Time Behavior of Solutions of Initial and Initial-Boundary Value Problems of a General System of Hyperbolic Conservation Laws*

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Abstract. We study the asymptotic behavior of the solution of the initial and initial-boundary value problem of hyperbolic conservation laws when the initial and boundary data have bounded total variation. It is shown that the solution converges to the linear superposition of traveling waves, shock waves and rarefaction waves. The strength and speed of these waves depend only on the values of the data at infinity.

§ 1. Introduction

We consider a system of conservation laws

$$U_t + F(U)_x = 0, \tag{1.1}$$

where $F(U)$ and U are n-vectors, $F=(F_1, \ldots, F_n)$, $U=(U_1, \ldots, U_n)$, $x \in R$ and $t \geqq 0$. We assume that the system is *strictly hyperbolic* and each characteristic field is either *genuinely-non-linear* or *linearly degenerate* in the sense of Lax [10]. We study the Cauchy problem (1.1) with initial data

$$U(x,0) = U_0(x) \tag{1.2}$$

which is assumed to have *bounded total variation* so that the limiting values of U_0 at $x = \pm\infty$ exist:

$$U_l \equiv U_0(-\infty), \qquad U_r \equiv U_0(+\infty).$$

Our main purpose is to compare the solution $U(x,t)$ of (1.1), (1.2) with the solution $U_*(x,t)$ of the corresponding Riemann problem (1.1) with

$$U(x,0) = \begin{cases} U_l & \text{for} \quad x<0, \\ U_r & \text{for} \quad x>0. \end{cases} \tag{1.3}$$

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