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Metastable States of Quantum Lattice Systems

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Abstract. We extend the characterisation of metastability, as given in [1] for classical systems, to show that quantal lattice systems with suitable long range forces can support metastable states.

In a previous work [1], we characterised metastable states of classical systems as being locally, but not globally, thermodynamically stable and also stable under constraints confining the system to a suitable reduced state space. It was shown that, according to this characterisation (whose motivation and relation to other ones [2, 3] was discussed in [1]), systems with certain types of long range forces can support metastable states. The object of this note is to show that similar conclusions are applicable to quantal lattice systems (cf. the Proposition and following Comment below).

Our treatment is based on a definition of local thermodynamical stability of such systems, that was introduced in [4]. There it was shown that, for systems with short range forces, the conditions for local stability are equivalent to those of KMS. In the present note, we need to extend our definition of local stability to systems with long range forces.

We formulate the states, observables and forces of a system on a lattice, Γ $(=Z^\nu$, say) in a standard way (cf. [5]). Thus, denoting the set $\{\Lambda\}$ of finite point subsets of Γ by L, we define the C^*-algebra $\mathscr{A}$ of observables of the system as that generated by a family $\{\mathscr{A}(\Lambda)|\Lambda\in L\}$ of local C^*-algebras, where $\mathscr{A}(\Lambda)$ is a type-I factor of finite order that is isotonic with respect to Λ. The state space Ω is then the set of positive normalised linear functionals on $\mathscr{A}$. The set of translationally invariant elements of Ω will be denoted by $\bar{\Omega}$. The forces in the system will be taken to correspond to an interaction potential ϕ, which maps L into the self-adjoint elements of $\mathscr{A}$ in such a way that (i) $\phi(\emptyset)=0$; (ii) $\phi(\Lambda)\in\mathscr{A}(\Lambda)$; (iii) ϕ transforms covariantly w.r.t. space translations; and (iv)

$$|\phi|\equiv\sum_{0\in\Lambda}\frac{\|\phi(\Lambda)\|}{N(\Lambda)}<\infty\,, \tag{1}$$