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KMS Conditions and Local Thermodynamical Stability of Quantum Lattice Systems. II

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Abstract. We prove that local thermodynamical stability (LTS), as defined in [1], implies the KMS conditions in quantum lattice systems, without any assumption of translational invariance. This result, together with those of [1], establishes the equivalence between the LTS and the KMS conditions for such systems.

Section 1

In an article by Araki and the author [1], the concept of local thermodynamical stability (LTS) was defined for quantum lattice systems; and it was shown that, if the forces were suitably tempered, the LTS conditions were implied by, and in the case of translationally invariant states equivalent to, those of Kubo-Martin-Schwinger (KMS). In the present article, we prove that the LTS conditions imply those of KMS, without any assumption of translational invariance, and thus establish the following theorem.

Theorem 1. *The* LTS *and* KMS *conditions are mutually equivalent for quantum lattice systems, subject to the same assumptions on the interactions as in* [1].

Comment. It has already been observed (cf. Note following Definition 2.2 in [2]) that, for classical lattice and hard-core continuous systems, the LTS conditions are equivalent to those of Dobrushin-Lanford-Ruelle (DLR). Thus, in view of the above theorem, we now conclude that LTS ≡ KMS for quantum lattice systems, and ≡ DLR for classical lattice and hard-core continuous ones.

Our notation will be based on that of [1]. We take Γ to be the lattice on which the system is situated; here it suffices to consider Γ as a denumerably infinite point set. The family $\{\Lambda\}$ of finite point subsets of Γ will be denoted by L. The algebra of observables, $\mathscr{A}(\Lambda)$, for the region $\Lambda(\in L)$ will be assumed to be a finite-dimensional, type-I factor, that is isotonic with respect to Λ and commutes with $\mathscr{A}(\Lambda')$ if $\Lambda \cap \Lambda' = \emptyset$; and the C^*-algebra of observables, $\mathscr{A}$, for the system will be taken to be the norm completion of $\mathscr{A}_L \equiv \bigcup_{\Lambda \in L} \mathscr{A}(\Lambda)$. The state space, Ω, of the system will be