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Free Energy in Spin-flip Processes is Non-increasing

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Abstract. In this paper, we propose a new and shorter proof of the following fact: in a spin-flip process on $\{-1,+1\}^S$, where S is a countable set, the free energy is non-increasing.

Free energy is a well defined functional only for invariant measures under a convenient group of bijections of S. We formalize this with the notion of B-ameanability of S. This frame contains the usual example of Z^d under translations but also many nice lattices that are not groups under groups of isometries.

For invariant measures, except Gibbs ones, the free energy is strictly decreasing. Among invariant measures, the only stationary measures for the spin-flip process are therefore Gibbs measures. From this result we also deduce an ergodic theorem.

The first result on this subject was obtained by Holley [1] for a finite range potential on $\{-1,+1\}^{Z^d}$ and some extension by Higuchi, Shiga [2].

Let S be a countable set, $X=\{-1,+1\}^S$. If x belongs to S, σ_x is the coordinate function at x, τ_x is the homeomorphism of the compact group X defined by: for $\eta\in X$, $\sigma_y\tau_x(\eta)=\sigma_y(\eta)$ if $y\neq x$ and $\sigma_x\tau_x(\eta)=-\sigma_x(\eta)$. G is the group generated by the $(\tau_x, x\in S)$ (finite modifications).

For Λ finite in S and A in Λ, the cylinder (A, Λ) is the subset of X:

$$\{\eta/\sigma_x(\eta)=-1 \text{ if } x\in A \text{ and } \sigma_x(\eta)=+1 \text{ if } x\in\Lambda-A\}.$$

If $x\in\Lambda$, (A_x, Λ) is the cylinder $(A\Delta\{x\}, \Lambda)$.

Let B be a group of bijections of S and in a natural way a group of homeomorphisms of X. In the following, we shall assume the fundamental hypothesis on the action of B:

B is transitive on S.

There is a sequence Λ_n of finite parts of S with the property:

$$\forall x_0\in S, \forall F \text{ finite } \subset S, \lim_{n\to\infty} |x\in\Lambda_n, \exists b\in B, b(x_0)=x, bF\subset\Lambda_n|/|\Lambda_n|=1.$$

We say that S is B-amenable and that Λ_n is a B-ameaning sequence.