Commun. math. Phys. 54, 283—292 (1977)

Communications in
Mathematical
Physics

Construction of a Selfadjoint, Strictly Positive Transfer Matrix for Euclidean Lattice Gauge Theories

M. Lüscher

II. Institut für Theoretische Physik der Universität Hamburg,
D-2000 Hamburg 50, Federal Republic of Germany

Abstract. It is shown that physical positivity holds in Wilson's lattice gauge theories, i.e. transition probabilities between gauge invariant states are non-negative and the quantum mechanical Hamiltonian has real eigenvalues only.

I. Introduction

Ever since lattice gauge theories were proposed by Wilson [1] there was the question, wether the scheme will indeed yield an acceptable quantum field theory in the continuum limit. One of the required properties that does not obviously hold in the lattice theory is physical positivity[1]. In this paper we are going to explicitly construct the quantum mechanical space of states for euclidean lattice gauge theories. We will also derive a formula for the transfer matrix, i.e. the operator e^{-aH}, where H is the q.m. Hamiltonian and a is the lattice spacing.

Euclidean lattice gauge theories are defined as follows (for details, the reader is refered to Wilson's papers). We consider a cubic, four dimensional lattice, whose points will be labelled by four integer numbers $n=(n_0, n_1, n_2, n_3)$, $|n_0| \leqq M$, $|n_i| \leqq L$ ($i = 1, 2, 3$), thus giving a total of $(2M+1)(2L+1)^3$ sites. At each lattice point n there is attached a classical Dirac spinor ψ_n (the quark field) whose entries are elements of a Grassmann algebra (cp. Appendix). The gauge field $U(n, \mu)$ ($\mu = 0, 1, 2, 3$) sits on the links between the lattice sites. It is an element of the gauge group G, which is taken to be $SU(N)$. Correspondingly, the quark field ψ_n carries a colour index α, $\alpha = 1, ..., N$. To keep the reasoning as transparent as possible, we will assume that there are no flavour degrees of freedom. Our results are however true for the more general case aswell.

The dynamics of euclidean quark and gluon fields can be expressed in terms of their correlation functions (euclidean expectations, Schwinger functions):

$$\langle \varphi_1 \dots \varphi_m \rangle = Z^{-1} \int \mathscr{D}U \mathscr{D}\psi \mathscr{D}\bar{\psi} \varphi_1 \dots \varphi_m e^A . \tag{1}$$

[1] Osterwalder and Seiler have announced a result concerning this question [5]