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Ground State Representation of the Infinite One-dimensional Heisenberg Ferromagnet

II. An Explicit Plancherel Formula

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Abstract. In its ground state representation, the infinite, spin 1/2 Heisenberg chain provides a model for spin wave scattering, which entails many features of the quantum mechanical N-body problem. Here, we give a complete eigenfunction expansion for the Hamiltonian of the chain in this representation, for *all* numbers of spin waves. Our results resolve the questions of completeness and orthogonality of the eigenfunctions given by Bethe for finite chains, in the infinite volume limit.

1. Introduction

Let H be the self adjoint Hamiltonian corresponding to the ground state representation of the spin 1/2, infinite one-dimensional Heisenberg ferromagnet with nearest neighbor interactions. The operator H is reduced by a spin-wave number operator, and H restricted to the N spin-wave sector is unitarily equivalent in a natural way to a second difference operator $-\Delta_N$ with "sticky" boundary conditions acting in an l^2-space.

The purpose of this article is to prove the completeness of an *explicit* eigenfunction expansion of $-\Delta_N$, for *all* N i.e. *all* numbers of spin-waves. This result was announced in [1]. In addition, using the generalized eigenfunctions for $-\Delta_N$, we construct a complete set of commuting self adjoint projections $\{E_\beta(\Delta)\}$ which reduce $-\Delta_N$. Here the subscript β called the binding, describes the manner in which the N-spin waves are bound together into "complexes" (in Bethe's terminology [2]), and Δ is a Borel subset of a torus whose dimension depends on the number of complexes comprising β. Any two projective $E_\beta(\Delta)$, $E_{\beta'}(\Delta')$ are orthogonal for β and β' distinct or if $\beta=\beta'$, for Δ and Δ' disjoint.

In fact, the projections $\{E_\beta(\Delta)\}$ were already obtained in [4] in a slightly different representation by considering the thermodynamic limit and utilizing the Bethe solution in [2] for the finite volume eigenfunctions. But the questions of

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