Commun. math. Phys. 54, 219—236 (1977)

Communications in Mathematical Physics

On the Integrability of the Lie Algebra of the Conformal Group in Quantum Field Theory

Szymon Rabsztyn

Institute of Theoretical Physics, University of Wrocław, Wrocław, Poland

Abstract. It is shown that the infinitesimal conformal symmetry implies (in any quantum field theory which satisfies the Wightman axioms without invoking locality and global Poincaré symmetry) that there exists a uniquely defined unitary representation of the universal (∞-sheeted) covering group of the Minkowskian conformal group $SO_e(4,2)/_{\mathbb{Z}_2}$. Proof was obtained using sufficient conditions for the integrability of a representation of a Lie algebra given by [8].

Introduction

Many papers have been written about global representations of the conformal group in the quantum field theories.

It has been first shown by [1–4] that global conformal transformations for free fields or generalized free fields can be defined without violation of causality.

Then Lüscher and Mack [5] proved that global Euclidean conformal symmetry of the Schwinger functions (weak conformal symmetry) implies that there exists a unique representation of the universal (∞-sheeted) covering group of the Minkowskian conformal group $SO_e(4,2)/_{\mathbb{Z}_2}$. In the same paper it has been stated also that a weak conformal symmetry implies the infinitesimal conformal symmetry of the fields.

The aim of the present paper is to prove that the infinitesimal conformal symmetry of the fields implies (in any quantum field theory which satisfies the Wightman axioms without invoking locality and global Poincaré symmetry) that there exists a uniquely defined unitary representation of the universal (∞-sheeted) covering group of $SO_e(4,2)/_{\mathbb{Z}_2}$. Our method is different from that used in the paper by Lüscher and Mack.

The problem is investigated in the following way: In Section 1 we present some facts concerning the mathematical background of the integrability problem.

The definition of a conformal quantum field theory is given in Section 2.

Finally, Section 3 deals with the problem of the integrability of the Lie algebra of the conformal group defined in Section 2.

Some details are included in the Appendix.