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The Central Limit Theorem and the Problem of Equivalence of Ensembles

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Abstract. In this paper we show that the local limit theorem is a consequence of the integral central limit theorem in the case of a Gibbs random field ξ_t, $t \in Z^\nu$ corresponding to a finite range potential.

We apply this theorem to show that the equivalence between Gibbs and canonical ensemble is a consequence of the integral central limit theorem and of very weak conditions on decrease of correlations.

Introduction

The integral and local limit theorems for sums of random variables belonging to a random process with dependent values have been considered in a lot of papers on probability theory, see for example [1] and the literature there quoted and also some more recent papers [2, 3]. In connection with the developments of the theory of random fields and their applications to physics in the last years some papers appeared concerning the integral limit theorem [6, 14, 15], and the local limit theorem for random fields [5, 4, 7].

From the point of view of statistical physics these local theorems are interesting because they are strongly connected with the problem of the equivalence of canonical and grand canonical ensembles, if one considers, as we do in this article, this equivalence not only in the sense of equality between thermodynamical functions, but also in the sense of equality between all the correlation functions. The known proofs of the local limit theorems can be applied in particular cases and are founded on some special methods developed for studying the Gibbs random field. We use different techniques and emphasize the connection between local and integral limit theorems which up to now have never been pointed out.

The main result of this paper (§ 1.1) is that the integral limit theorem for an integer valued Markov field with non vanishing conditional probabilities (or, which is the same, [8] for a Gibbs random field with a range R finite and bounded potential) implies the local limit theorem.

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