Commun. math. Phys. 54, 151—172 (1977)

Communications in
Mathematical
Physics

A Generalization of the Classical Moment Problem on *-Algebras with Applications to Relativistic Quantum Theory. II.

Michel Dubois-Violette
Laboratoire de Physique Théorique et Hautes Énergies, Université de Paris XI, F-91405 Orsay, France*

Abstract. We discuss some properties of a non-commutative generalization of the classical moment problem (the m-problem) previously introduced. It is shown that there is a connexion between the determination of the problem and the self-adjointness properties in the corresponding Hilbert space. This generalizes the well-known connexion between the determination of the measure in the classical moment problem and the self-adjointness properties of the polynomials as operators in the corresponding L^2-space. The dependence of the m-problem on the choice of C^*-semi-norms and on the action of *-homomorphisms is also investigated. As an application, it is shown that if a quantum field (in a very general sense) is essentially self-adjoint then the m-problem for the Wightman functional is determined on the quasi-localizable C^*-algebra and that the corresponding representation of the localizable algebra generates the bounded observables of the field. It is pointed out that (ultraviolet and spatially) cut-off fields fall in this class and, therefore, are in one to one correspondance with states on the quasi-localizable C^*-algebra.

1. Introduction

This paper is a continuation of a preceding one [1] hereafter referred as Part I. Its object is to complete the algebraic discussion of the non-commutative generalization of the classical moment problem (the m-problem) introduced in Part I and to extend the applications to quantum field theory.

Let us first describe an important result on the classical moment problem [3–5] which will be generalized in this paper. Let ϕ be a positive linear form on the *-algebra $\mathbb{C}[X]$ of the complex polynomials with respect to one indeterminate X. Basically, to solve the moment problem for ϕ means to produce a self-adjoint operator $\pi(X)$ in a Hilbert space $\mathfrak{H}$ with a vector $\Omega \in \bigcap_{n \geq 0} \mathrm{dom}(\pi(X)^n)$ in such a way

---

* Laboratoire associé au Centre National de la Recherche Scientifique. Postal address: Laboratoire de Physique Théorique et Hautes Énergies, Bâtiment 211, Université de Paris-Sud, F-91405 Orsay, France