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Application of Commutator Theorems to the Integration of Representations of Lie Algebras and Commutation Relations*

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Abstract. Sufficient conditions on unbounded, symmetric operators A and which imply that

$$\exp(itA)\exp(isB)\exp(-itA)$$

satisfies the well known "multiple commutator" formula are derived. Th formula is then applied to prove new necessary and sufficient conditions for tl integrability of representations of Lie algebras and canonical commutatic relations and the commutativity of the spectral projections of two commutin unbounded, self-adjoint operators. A classic theorem of Nelson's is obtained ε a corollary. Our results are useful in relativistic quantum field theory.

1. Introduction

In this note we discuss sufficient conditions for the multiple commutator formu

$$e^{itA}e^{isB}e^{-itA} = \exp is\left\{B + \sum_{n=1}^{\infty} \frac{(it)^n}{n!} \operatorname{ad}^n A(B)\right\}, \quad (1.$$

to hold. Here A and B are unbounded operators and, formally,

$$\operatorname{ad} A(B) = [A, B], \quad \operatorname{ad}^n A(B) = [A, \operatorname{ad}^{n-1} A(B)]. \quad (1.$$

Our results have applications in group theory and quantum field theory.

They are a direct outgrowth of recent work of Driessler and the author [2 concerning the Haag-Kastler axioms [12] in relativistic quantum field theory and subsequent alternate proof of the main result of [2] due to Glimm and Jaffe [3

The main result of [2, 3], a sufficient condition for the bounded functions of tw unbounded, symmetric operators A and B to commute, is a special case of tł results proven in the following sections.

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