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Coherent State Representations of Nilpotent Lie Groups

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Abstract. The coherent state representations of a connected and simply connected nilpotent Lie group are characterized in terms of the Kirillov correspondence, as being those irreducible unitary representations whose associated orbits under the coadjoint representation are linear varieties.

Introduction

The concept of coherent states, originally related to the Weyl representation of the Heisenberg group, has been extended by Perelomov [8] to a general group theoretical setting, involving an irreducible unitary representation of an arbitrary Lie group. We shall adopt here a slightly modified version of Perelomov's definition, which reveals from the very beginning the classical phase space that parametrizes the coherent states. Our concern in this paper is to determine which irreducible unitary representations of an arbitrary nilpotent Lie group admit coherent states. The interest for nilpotent groups is motivated not only by the fact that the Heisenberg group belongs to this class, but also by the existence of the Kirillov orbital description for the unitary dual of such a group. Indeed, we find that, in terms of the Kirillov correspondence, the irreducible unitary representations of a connected and simply connected nilpotent Lie group which admit coherent states are associated with those orbits of its coadjoint representation which are of the simplest geometric form, namely linear varities. In other words, they are essentially square integrable representations, like in the special case of Heisenberg groups. Moreover, we show that, in a certain sense, the only classical phase space which may support a system of coherent states for such a representation is precisely the corresponding orbit.

1.

Let G denote a locally compact group and X a G-homogeneous space which admits an invariant measure μ_X. After fixing a point $0 \in X$, we shall identify X to the quotient space G/G_0, where G_0 is the isotropy subgroup of G at 0, and the measure