

Trace States and KMS States for Approximately Inner Dynamical One-Parameter Groups of *-Automorphisms

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Abstract. The usual definition of approximately inner one-parameter groups of *-automorphisms of C^* -algebras (approximately inner dynamical one-parameter groups) contains a slight asymmetry. When this asymmetry is “corrected”, we show that if an approximately inner dynamical one-parameter group has *KMS* states for *one* value of inverse temperature $\beta = 1/kT$, then it has *KMS* states for all values of β . By the Powers-Sakai Theorem it is enough to show that there is a trace state. We obtain a trace state as a limit of a sequence of “vector states” with respect to a given *KMS* state and thus solve a problem raised in [6].

1. Introduction

In the global and axiomatic approach to statistical mechanics [12] the algebra of observables is a C^* -algebra \mathfrak{A} with unit, and the dynamics $\{\alpha^t: -\infty < t < \infty\}$ is given by a strongly continuous one-parameter group of *-automorphisms of \mathfrak{A} . The dynamical group α^t is said in [9] to be approximately inner if there exists a sequence of hermitian elements (h_n) in \mathfrak{A} such that for all $a \in \mathfrak{A}$ and $t \in \mathbb{R}$ we have

$$(1) \quad \|e^{ith_n} a e^{-ith_n} - \alpha^t(a)\| \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

If α^t is assumed strongly continuous, then it follows from the Trotter-Kato Theorem ([3, 15] and [17]) that the convergence in (1) is uniform for t in compact subintervals of the real line. If on the other hand the convergence is assumed uniform for t in compact intervals, then α^t is automatically strongly continuous, as can be checked by a 2ε argument.

We suggest adding one minor technical condition to the definition of approximately inner one-parameter groups in order to make the definition symmetric in a certain sense. (In known examples, the additional condition is automatically satisfied.) Consider the following possible additional properties:

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