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Extension of KMS States and Chemical Potential

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Abstract. We present an algebraic description of the concept of chemical potential for a general compact gauge group G, as a first step in the classification of thermodynamical equilibrium states of a given temperature. Adopting first the usual setting of a field algebra $\mathfrak{F}$ containing the observable algebra $\mathfrak{A}$ as its gauge invariant part, we establish the following results (i) the existence and uniqueness, up to gauge, of τ-weakly clustering states ϕ of $\mathfrak{F}$ extending a given such state ω of $\mathfrak{A}$ (τ an asymptotically abelian automorphism group of $\mathfrak{F}$ commuting with G) (ii) in the case of an ω faithful and β-KMS for a time evolution commuting with τ, and of a time-invariant ϕ, the fact that ϕ is β-KMS for a one-parameter group of time and gauge whose gauge part lies in the center of the stabilizer G_ϕ of ϕ. (iii) a description of the general case where ϕ is neither time invariant nor faithful: ϕ is then in general vacuum-like in directions of the gauge space governed by an "asymmetry subgroup". We further analyze the representations and von Neumann algebras determined by ω, ϕ and the gauge average $\bar{\omega}$ of ϕ. The covariant representation generated by $\bar{\omega}$ is shown to be obtained by inducing up from G_ϕ to G the representation generated by ϕ. Finally we present, for the case where G is an n-dimensional torus, an intrinsic description of the chemical potential in terms of cocycle Radon-Nicodym derivatives of the state ω w.r.t. its (quasi equivalent) transforms by localized automorphisms of $\mathfrak{A}$. Our main result (ii) is established using two independant techniques, the first making systematic use of clustering properties, the second relying on the analysis of representations. Both proofs are basically concerned with Tannaka duality—the second with a version thereof formulated in Robert's theory of Hilbert spaces in a von Neumann algebra.

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