Commun. math. Phys. 53, 65—96 (1977)

Communications in Mathematical Physics



# Hilbert Space Sectors for Solutions of Non-linear Relativistic Field Equations

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**Abstract.** In the set of Cauchy data corresponding to the solutions of non-linear classical relativistic field equations having locally finite kinetic energy a structure of Hilbert space sectors is introduced. Each sector is invariant under time evolution and a total energy and linear momentum functionals can be defined as global quantities. Within this framework the existence of conserved dynamical charges is proved and the mechanism by which a symmetry can be spontaneously broken is explained.

## 0. Introduction

Recently there has been a revived interest in solutions of classical non-linear field equations as a means of understanding basic properties of elementary particles [1]. The main idea is to classify some simple stable solutions of field equations and to analyze small perturbations around them. Examples of such solutions are the constants which minimize the energy [2] and the solitons [3]. One hopes that essential features coming from the non-linear character of the theory are taken care of by the structure of those special solutions. This approach to non-linear field theory is also crucial for understanding spontaneous symmetry breaking [2], stability problems and for explaining the occurrence of charges which are of dynamical rather than of group-theoretical origin [4]. Moreover the properties of classical solutions of non-linear field equations are relevant for the quantum field theory version since they correspond to the expectation values of quantum fields on suitable coherent states [5]. These ideas look interesting and promising and deserve a systematic investigation, which seems to be lacking in the literature.

The purpose of this paper is to provide a rigorous treatment of some non-linear systems of partial differential equations for classical fields along the above lines. We shall be able to answer a certain number of questions like the construction

* Partially supported by C.N.R. (gruppo G.N.A.F.A.)