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Two-dimensional Neutral Coulomb Gas

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Abstract. Accurate bounds for the classical canonical partition function of the two-dimensional Coulomb gas interacting through the Coulomb potential $-q_i q_j \log r_{ij}$ are calculated (valid for all $T > T_c$). The existence of the thermodynamic limit is proved.

In a recent paper Deutsch and Lavaud [1] speculated about the thermodynamic limit for the two-component classical neutral Coulomb gas in two-dimensions, i.e. the existence of finite thermodynamic functions per particle in the limit $N \to \infty$, $V \to \infty$ with $\frac{N}{V}$ bounded.

Their estimates were not accurate enough to prove the existence of the thermodynamic limit. Recently Fröhlich [12] has solved the problem using euclidean quantum field techniques (Gaussian integration).

The objective of this paper is to obtain stronger bounds for the canonical partition functions of the Coulomb gas and then prove the existence of the thermodynamic limit using well established statistical mechanics techniques.

The general scheme of the paper is, by sections, as follows:

1. Definitions. The partition function, free energy per particle, etc. are defined and the problem of the thermodynamic limit precisely stated.

2. Upper bound for classical partition function.

3. Basic inequality for classical systems. An inequality relating the canonical partition function for domain Ω to the partition function for two sub-domain s Ω' and Ω'' is obtained.

4. Limit for cubes. A standard sequence Γ_k of cubic domains, each double the size of its predecessor is introduced. The existence of the limiting free energy is established for this sequence by showing (essentially) that the free energy per particle is monotonically increasing in k and recalling the upper bound (Section 2).

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