

## Implementation of Automorphisms and Derivations of the CAR-Algebra

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**Abstract.** The implementability of automorphisms and derivations of the CAR-algebra in a pure quasifree state is discussed in detail. Especially the properties of the implementing operators are investigated, and an explicit construction is given. Extending a result from [2] for the CCR-algebra, we get a new necessary condition for a derivation to be implementable by a selfadjoint operator.

### 1. Introduction

Whereas there are many results about the implementability of automorphisms and derivations of the form  $\psi(f) \rightarrow \psi(Sf)$ , little is known about the properties of the implementing operators and their construction. These questions can be answered by applying the methods of constructive quantum field theory. Some useful and interesting new results come out. Among them there are: Estimates, corresponding to the  $N_\tau$ -estimates in constructive quantum field theory, criteria for selfadjointness and an explicit construction of a bilinear form which implements a given automorphism. If this automorphism satisfies the criterion of Shale and Stinespring [3], the implementing form can be extended to a unique bounded operator, which differs from a unitary only by a constant factor. If the automorphism does not map any annihilation operator on a pure creation operator, the bilinear form can be given in a closed form, in which it can be compared with the formal expression resulting from the linked cluster theorem.

In the last section a new necessary condition is found for a derivation to be implementable by a selfadjoint operator, analog to Hochstenbach's result in the scalar field case [2]. It proves the conjecture that the first term in the perturbation series decides over the implementability. It follows the explicit construction of the implementing operators with the method of dressing transformations and a comparison of this method with approximation techniques.

The results presented in this paper are essentially contained in the authors thesis [1]. Some of these results have been obtained also by Ruijsenaars [4].