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Symmetry and Equilibrium States

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Abstract. Within the general framework of C^*-algebra approach to mathematical foundation of statistical mechanics, we prove a theorem which gives a natural explanation for the appearance of the chemical potential (as a thermodynamical parameter labelling equilibrium states) in the presence of a symmetry (under gauge transformations of the first kind). As a symmetry, we consider a compact abelian group G acting as *-automorphisms of a C^*-algebra $\mathfrak{A}$ (quasi-local field algebra) and commuting (elementwise) with the time translation automorphisms ϱ_t of $\mathfrak{A}$. Under a technical assumption which is satisfied by examples of physical interest, we prove that the set of all extremal ϱ_t-KMS states φ (pure phases) of G-fixed-point subalgebra $\mathfrak{A}^G$ (quasi-local observable algebra) of $\mathfrak{A}$ satisfying a certain faithfulness condition is in one-to-one correspondence with the set of all extremal G-invariant $\varrho_t \cdot \alpha_t$-KMS states φ^- of $\mathfrak{A}$ with α varying over one-parameter subgroups of G (the specification of α being the specification of the chemical potential), where the correspondence is that the restriction of φ^- to $\mathfrak{A}^G$ is φ.

§ 1. Introduction

In equilibrium statistical mechanics, one of basic problems is how to understand the number of independent thermodynamic variables for equilibrium states of a given system. In the Heisenberg picture, a mathematical description of a system is given in terms of observables and its time translation. So-called KMS condition picks out states labelled by a real parameter β; they are interpreted as equilibrium states at inverse temperature β. The justification for such an interpretation has been given by Haag et al. [2], who characterize KMS states by stability under dynamical perturbation and then show that the stability condition is equivalent to the KMS condition under some assumptions.

For given β, there may be many KMS states which can be labelled by additional macroscopic variables. It is also useful to consider a family of time-translations labelled by some external parameters (such as external magnetic