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Critical Exponents and Elementary Particles

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Abstract. Particles are shown to exist for a.e. value of the mass in single phase ϕ^4 lattice and continuum field theories and nearest neighbor Ising models. The particles occur in the form of poles at imaginary (Minkowski) momenta of the Fourier transformed two point function. The new inequality $dm^2/d\sigma \leqq Z$, where $\sigma = m_0^2$ is a bare mass^2 and Z is the strength of the particle pole, is basic to our method. This inequality implies inequalities for critical exponents.

1. Introduction

Euclidean ϕ^4 fields are believed to describe the asymptotic long distance behavior of certain lattice models of statistical mechanics at their critical points (e.g. the Ising model). It was proposed [9] that a construction of ϕ^4 fields could be based on this expectation, and partial results in this direction are given in [6–8, 10, 11, 1, 12]. In this construction, the field ϕ will be nontrivial (i.e. not a free field) only in the case in which the corresponding lattice model critical point is asymptotically nontrivial at long distances.

In order to better distinguish between the trivial and the nontrivial cases, we continue here our investigation [7, 10, 11] of critical exponents (see also § 5). In general, our results have the form

$$\text{canonical exponent} \leqq \text{general exponent}, \tag{1.1}$$

$$\phi\text{-exponent} \leqq \phi^2\text{-exponent}, \tag{1.2}$$

and in particular if the lattice ϕ^2 field (e.g. the Ising model energy-energy correlation) is canonical, then so is the corresponding lattice ϕ-field and also the resulting continuum ϕ-field. The converse to this statement seems to be false, and a counterexample may be found in the ϕ_4^4 lattice field at weak coupling. In this model, there is some evidence that ϕ^2 deviates from canonical by a logarithm.

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