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Decomposition of Many-Body Schrödinger Operators

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Abstract. The complex-dilated many-body Schrödinger operator $H(z)$ is decomposed on invariant subspaces associated with the "cuts" $\{\mu+z^{-2}R^{+}\}$, where μ is any threshold, and isolated spectral points. The interactions are dilation-analytic multiplicative two-body potentials, decaying as $r^{-1+\varepsilon}$ at $r=0$ and as $r^{-\varepsilon}$ at $r=\infty$.

Introduction

The non-relativistic quantum mechanical many-body problem has been the object of several mathematical investigations during recent years. One of the central problems is that of asymptotic completeness for systems with multichannel scattering. Since the work of Faddeev [4] on the 3-body problem new major steps have been taken by Ginibre and Moulin [5] and Thomas [11] who have independently generalized Faddeev's result in the 3-body case and at the same time simplified the proofs, utilizing different, though related Hilbert space methods. Another important work is due to Sigal [9] who has extended Faddeev's result to the n-body case under essentially the same assumptions and utilizing the same mathematical tools, working with Banach spaces of Hölder-continuous functions.

A different approach to the n-body problem was introduced by the author and Combes [3] and independently by van Winter [12]. This may be called the analytic theory of many-body Schrödinger operators, with the dilation-analytic version of [3] and the complex dynamical variables version of [12]. The basic feature of this theory is the construction of a selfadjoint analytic family of operators $H(z)$ in an angle $S_a=\{z=\varrho e^{i\varphi}|\varrho>0|\varphi|<a\}$, such that $H(1)=H$. The operators $H(\varrho e^{i\varphi})$, $\varphi \neq 0$, although non-selfadjoint, are more accessible for analysis than H itself, whose properties can then be derived from those of $H(e^{i\varphi})$ by letting $\varphi\to 0$. This program was carried out in [3], so far as the spectral properties are concerned, for a large class of interactions. When the imaginary parameter φ is "turned on", the essential spectrum rotates to the angle -2φ and splits up into a system of