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# Dimensional Renormalization and the Action Principle

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**Abstract.** Dimensional renormalization is defined in such a way that the renormalized action principle holds. It is shown that this leads to a minimal, additive renormalization. The derivation of Ward-Takahashi indentities and Callan-Symanzik equations from the action principle is exemplified.

## I. Introduction

Dimensional regularization [1, 2] has become an almost indispensable tool to the perturbative treatment of non-abelian gauge theories. Nevertheless the first complete proof of the so-called Slavnov-Taylor identities, guaranteeing the unitarity of the $S$-matrix, has been given recently with help of the "Normal Product Algorithm" (NPA) based on BPHZ renormalization [3]. The reason lies in the validity of the renormalized action principle in that formalism. The action principle specifies the change of the Green's functions under an infinitesimal variation of fields or parameters entering the Lagrangian. In our opinion it provides a much more efficient approach to the proof of Ward-Takahashi or Slavnov-Taylor identities—or other structural properties—than the algebraic manipulations performed on individual diagrams advocated by the pioneers in that field [4]. The approach of Ref. [3] suffers, however, from the fact that the subtraction method in the BPHZ renormalization is not compatible with the identities to be proved. This is overcome by appropriately chosen asymmetric finite counterterms added to the Lagrangian. Apart from the difficulty to prove that such counterterms can really be found in all orders of perturbation theory the method appears much too clumsy for practical calculation.

Therefore, it is only natural to make an effort to establish the renormalized action principle in the framework of dimensional renormalization. This is one of the main objectives of the present work. The second is to show that dimensional renormalization meets the requirements of a minimal, additive renormalization

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