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# Two Bianchi Type VIII Spatially Homogeneous Cosmologies*

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**Abstract.** We study two Bianchi type VIII analogues of Taub space and maximal analytic extensions of them. The first one has SL(2, $R$) as an isometry group, which acts transitively on spacelike hypersurfaces. The maximal extension has all of the pathological features of Taub-NUT space. The second one has the universal covering group of SL(2, $R$) as an isometry group. The maximal extension of the latter does not have these pathological properties and is geodesically complete.

## 1. Local Solutions of the Einstein-Maxwell Equations

Local solutions of the Einstein-Maxwell vacuum field equations have been derived by Carter [1] under the condition that the metrics admit a two-dimensional abelian isometry group and that the Hamilton-Jacobi equation for the geodesics and the Schrödinger equation separate in certain coordinate systems. The metrics contain several parameters and when some of the parameters are zero, the metrics admit a four-dimensional local isometry group. It is our aim to study two of these metrics that admit a four-dimensional local isometry group and show that they are special Bianchi type VIII spatially homogeneous cosmologies. Bianchi's classification of three-dimensional real Lie algebras into nine types is given by Taub [2]. Since the two metrics are Bianchi type VIII analogues of the Bianchi type IX metric discovered by Taub [2], we first review the known facts about the latter metric [3]. We will compare Taub space with the two Bianchi type VIII metrics throughout the paper.

The three metrics under consideration in this paper are all special cases of Carter's [1] metrics listed in class $[\tilde{B}(+)]$. We first consider the "charged" Taub-Nut metric [4] given by

$$g=(t^2+l^2)(d\theta^2+\sin^2\theta d\phi^2)-4l^2\Delta(t^2+l^2)^{-1}(d\psi+\cos\theta d\phi)^2 \\ +(t^2+l^2)\Delta^{-1}dt^2\,, \qquad (1)$$

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