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Percolation and Phase Transitions in the Ising Model

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Abstract. We give a description of the mechanism of phase transitions in the Ising model, pointing out the connection between the spontaneous magnetization and the existence of infinite clusters of "up" and "down" spins. The picture is more complete in the two-dimensional Ising model, where we can also use a generalized version of a result by Miyamoto.

1. Introduction

Percolation problems have been mostly studied for non-interacting systems (for a general review, see for example [1]). Only recently other cases have been considered: rigorous results are proved in [2], where site percolation problems for Ising spins on Bethe lattices are solved, and in [3], where Miyamoto extends to a class of interacting systems a classical result stated by Harris [4] for the random bond percolation problem on the plane square lattice.

In this paper we consider only site percolation problems, because in our picture they are more strictly related to the Ising model than the bond ones.

In Section 2 we consider the ν-dimensional Ising model. We prove that, at zero external field and for $T < T_c$, percolation probability and spontaneous magnetization are related by an inequality.

We next limit ourselves to the case $\nu = 2$. We first give a further extension of the theorem proved in [3] under the condition of "symmetry of configuration", observing that it has a natural generalization to the non-symmetric cases. Furthermore, we prefer to reformulate the statement for the site percolation problem using the matching graph of the plane square lattice rather than its dual graph. This is done in Section 3.

Finally, in Section 4 we simultaneously use the results of the preceding sections. We show that, at zero external field, in the single phase region there are no infinite clusters, while in the two phases region each pure phase is characterized by the existence of an infinite cluster of the corresponding sign.

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