

Solution of the Local Mass Problem in General Relativity

Yvonne Choquet-Bruhat

Département de Mécanique, Université de Paris VI, F-Paris, France

Jerrold E. Marsden*

Department of Mathematics, University of California, Berkeley, California 94720, USA

Abstract. The local mass problem is solved. That is, in suitable function spaces, it is shown that for any vacuum space-time near flat space, its mass m is strictly positive. The relationship to other work in the field and some discussion of the global problem is given. Our proof is, in effect, a version of critical point analysis in infinite dimensions, but detailed L^p and Sobolev-type estimates are needed for the precise proof, as well as careful attention to the coordinate invariance group. For the latter, we prove a suitable slice theorem based on the use of harmonic coordinates.

0. Introduction

For some time there has been controversy over the definition and positivity of the mass of an asymptotically euclidean (vacuum) solution of Einstein's equations.

Brill [3] established positivity for time-symmetric and axial-symmetric spacetime. Araki [1] proved the positivity of the second variation of the Schwarzschild mass of a certain class of time-symmetric solutions constructed by conformal methods. Finally Brill and Deser [4] outlined a proof of positivity in the general case.

The method of Brill and Deser is to show that the mass function has only one critical point, namely at flat space and that the second variation is strictly positive there. The proof is, however, incomplete for four reasons.

First of all, they assumed the existence of maximal slices (i.e., slices whose second fundamental form has zero trace), which was open to question. Secondly, the topology in which the second variation is positive definite is not the same as the topology on the initial data set, so that it is far from obvious that flat space is a local minimum. Thirdly, the problems connected with the coordinate invariance group require attention on the space of initial data as well as on an infinitesimal level (the quotient space may well be singular). Fourthly, the global

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