Commun. math. Phys. 51, 243—282 (1976)

Communications in
Mathematical
Physics

Nonextendible Positive Maps

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Abstract. Positive maps of ordered vector spaces into the algebra of all bounded operators acting on a Hilbert space are considered. A special class of so called nonextendible maps is introduced and investigated. This class is much smaller than the class of extreme maps.

Any positive map can be obtained from a nonextendible one by restriction.

In the C^*-algebra case, the nonextendibility of a normalized positive map ϕ is related to the properties of the expression $\phi(a^2)-\phi(a)^2$. In particular Jordan representations are non-extendible.

2-positive nonextendible maps are representations. Similar result holds for copositive maps. For abelian C^*-algebras, notion of nonextendible map and that of representation coincide.

The nonextendible positive maps of the Jordan algebra M_{2s} of all 2×2 symmetric matrices and of the full 2×2 matrix algebra are especially investigated. Any nonextendible normalized positive map of M_{2s} is a Jordan representation. M_2 admits nonextendible normalized positive maps not being Jordan representations. A large class of examples is given.

0. Introduction

Let $\mathfrak{A}$ and $\mathfrak{B}$ be C^*-algebras. We denote by $\mathfrak{A}_+$ and $\mathfrak{B}_+$ the cones of positive elements and by $1_{\mathfrak{A}}$ and $1_{\mathfrak{B}}$ the unity elements of these algebras. We shall consider linear maps ϕ of $\mathfrak{A}$ into $\mathfrak{B}$ such that $\phi(\mathfrak{A}_+)\subset\mathfrak{B}_+$ and $\phi(1_{\mathfrak{A}})=1_{\mathfrak{B}}$. To stress these properties we write

$$\phi:(\mathfrak{A},\mathfrak{A}_+,1_{\mathfrak{A}})\to(\mathfrak{B},\mathfrak{B}_+,1_{\mathfrak{B}})\,. \tag{0.1}$$

We say that ϕ is a normalized positive map. In recent years, positive maps have become of common interest to mathematicians and physicists in particular in connection with the operator theory (cf. [1]) and the quantum theory of open systems (cf. [6, 8, 12]). The notion of positive map generalizes that of state, re-

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