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On the Construction of Quasimodes Associated with Stable Periodic Orbits*

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Abstract. Let $H(x, D, \varepsilon)$ be a self-adjoint partial differential operator of the form

$$H = \sum_{k=0}^{K} \varepsilon^k H_k(x, \varepsilon D), \qquad x \in R^n.$$

Suppose the hamiltonian system

$$\dot{x} = \frac{\partial H_0}{\partial \xi}, \qquad \dot{\xi} = -\frac{\partial H_0}{\partial x}$$

has a nondegenerate stable periodic orbit γ on which $\dot{x} \neq 0$. Then it is possible to construct a sequence of real numbers ε_m tending to zero, a sequence of functions u_m concentrated in a tube of radius $\varepsilon_m^{1/2}$ about the projection of γ into x-space, and a polynomial $E(\varepsilon)$ such that

$$\|(H(\varepsilon_m) - E(\varepsilon_m))u_m\| \leqq C\varepsilon_m^M \|u_m\|.$$

The power M depends on the order of stability of γ. The constructions are explicit in terms of solutions of linear O.D.E.'s, and are generalizations of "gaussian beams". Actually, instead of just one sequence, one gets a family of sequences parametrized by the multi-indices of order $n-1$, but the constant C is not independent of these multi-indices. The nondegeneracy hypothesis implies γ is part of a one-parameter family of stable periodic orbits, and C is independent of this parameter.

After presenting the constructions, we discuss their application to the quasi-classical limit in quantum mechanics and their relation to work of Keller, Maslov and others.

We wish to study the behavior of the spectrum of a linear partial differential operator, $P(x, D, \varepsilon)$, depending on a parameter ε, as ε tends to zero.. We assume

$$P(x, D, \varepsilon) = \sum a_\alpha(x, \varepsilon) D^\alpha$$

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