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Field Theory with an External Potential

W. J. M. A. Hochstenbach
Institut für Theoretische Physik, Universität Göttingen, D-3400 Göttingen,
Federal Republic of Germany

Abstract. A quantum theory for charged spin zero particles interacting with an external potential is constructed for a certain class of time-independent potentials.

For potentials of a different class, a group of Bogoliubov transformations generated by the solutions of a classical differential equation with an external potential is defined in the free one-particle space. We give necessary and sufficient conditions on the potential for this group to be unitarily implementable in the Fock space of the free field.

1. Introduction

Although perturbation theory constitutes the basis for the practical calculations in quantum electrodynamics, a direct justification for applying perturbation theory is prevented by the existence of a Euclidean symmetry group. Generally one tries to circumvent this implication of Haag's theorem by breaking the symmetry.

For charged spin zero particles in an external time-independent potential we prove in Section 3 a conjecture of Schroer, Seiler and Swieca [1]: the interaction Hamiltonian does not exist in the Fock space of the free field if the external potential contains a three-vector part. Thus, in a field theory with an external, time-independent vector potential $(0, \boldsymbol{A})$ the assertion of Haag's theorem is valid, although its assumptions are not fulfilled.

To be more precise, let (A^μ) $(\mu=0,1,2,3)$ be an element of the function class $\mathscr{Q}$ specified in Section 3; the time evolution of the interacting field in the Fock space of the free particles is determined by a one-parameter group of Bogoliubov transformations in the free one-particle space; at time $t=0$ the interacting field coincides with the free field. Then the time evolution is described by a strongly continuous, one-parameter group of unitary operators in the Fock space of the free field, if and only if $\boldsymbol{A}=0$ on $\mathbb{R}^3$ and

$$Q_0(t)=\int \frac{d\boldsymbol{p}d\boldsymbol{p}'}{\omega(\boldsymbol{p})\omega(\boldsymbol{p}')}\left(\frac{\omega(\boldsymbol{p})-\omega(\boldsymbol{p}')}{\omega(\boldsymbol{p})+\omega(\boldsymbol{p}')}\right)^2 |\tilde{A}_0(\boldsymbol{p}-\boldsymbol{p}')|^2 \sin^2\frac{(\omega(\boldsymbol{p})+\omega(\boldsymbol{p}'))t}{2}$$

is finite for all times t and continuous in t at 0.