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On Perturbations of the Periodic Toda Lattice

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Abstract. A class of Hamiltonian systems including perturbations of the periodic Toda lattice and homogeneous cosmological models is studied. Separatrix approximation of oscillation regimes in these systems connected with Coxeter groups is obtained. Hamiltonian systems connected with simple Lie algebras are pointed out, which generalize the system describing periodic Toda lattice and allow the $L-A$ pair representation.

1. Introduction and Summary

The Toda lattice is known [1–5] to be an infinite system of unit mass particles, whose interaction is determined by the potential

$$V=\sum_i \exp(q_i-q_{i+1}),$$

where q_i is the displacement of the i-th particle from the equilibrium. The periodic Toda lattice is determined by $q_{i+n+1}\equiv q_i$ condition and has the Hamiltonian

$$H=1/2\sum_{i=1}^{n+1} p_i^2+\sum_{i=1}^{n}\exp(q_i-q_{i+1})+\exp(q_{n+1}-q_1). \tag{1.1}$$

In the present paper we study the Hamiltonian systems generalizing (1.1):

$$\dot{p}_i=-\partial H/\partial q_i, \qquad \dot{q}_i=\partial H/\partial p_i$$

$$H=1/2\sum_{i,j}^{n} a_{ij}p_ip_j+\sum_{k,m}^{n+1} \mathscr{b}_{km}(\{\alpha_k, q\}+\{\alpha_m, q\}). \tag{1.2}$$

Here $\alpha_1, \ldots, \alpha_{n+1}$ are vectors in n-dimensional space R^n with coordinates $\alpha_k=(d_{k1}, \ldots, d_{kn})$, q being the vector $(q_1, \ldots, q_n)$. In R^n two scalar products (x, y) and $\{x, y\}$ are given:

$$(x, y)=\sum_{i,j}^{n} a_{ij}x_iy_j, \qquad \{x, y\}=\sum_{i=1}^{n} x_iy_i.$$