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Non-radiative Solutions of Einstein's Equations for Dust

W. B. Bonnor

Department of Mathematics, Queen Elizabeth College, University of London, London W8 7AH, England

Abstract. The quasi-spherical collapsing space-time of Szekeres is investigated. The arbitrary functions can be chosen so that it has positive density, and no Killing vectors; yet a ball $r<r_0$ of it can joined to the Schwarzschild metric, and is therefore non-radiative.

1. Introduction

Whether freely falling bodies radiate gravitational waves is an old question, dating back to the famous work of Einstein, Infeld and Hoffmann on motion. (For a review of early work see [1].) This question has certainly not yet been answered, as has recently been emphasised by Ehlers et al. [2]. Although several approximate methods have shown convincingly that in certain motions of bodies gravitational waves are produced, *these methods do not apply to free fall;* and approximation methods specially designed for free fall have not yet produced satisfactory results [2].

An answer could be provided by appropriate exact solutions of Einstein's equations for dust, which is a good model for freely falling matter. Until 1975 the known exact solutions for dust, all of which had high spatial symmetry, seemed, with one possible exception, not to contain gravitational radiation, though it is hard to test whether radiation is present or not. The exception was the case of cylindrical symmetry for which Cocke [3] claimed to find an energy transmission corresponding to gravitational waves. I am not convinced by this work for two reasons. First, the concept of energy in the cylindrical case has not been satisfactorily formulated, partly because the space-time is not Minkowskian at infinity; secondly, since Cocke's cylinders are infinite in length, some interference with the energy balance by an influx at the ends cannot be ruled out. Thus we can say that exact solutions up to 1975 gave an inconclusive answer to our question.

In 1975 Szekeres [4, 5] published an exact solution which is much more promising, and which will be investigated in this paper. The basic idea of this work is to show that a portion of the solution having finite spatial volume can be joined to a Schwarzschild metric. Since the latter is static, it follows that, whatever