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Generalized Trotter's Formula and Systematic Approximants of Exponential Operators and Inner Derivations with Applications to Many-Body Problems

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Abstract. New systematic approximants are proposed for exponential functions, operators and inner derivation δ_H. Remainders of systematic approximants are evaluated explicitly, which give degrees of convergence of approximants. The first approximant corresponds to Trotter's formula [1]: $\exp(A+B)=\lim_{n\to\infty}[\exp(A/n)\exp(B/n)]^n$. Some applications to physics are also discussed.

1. Introduction

In this paper, we investigate systematic approximants and errors of exponential operators such as e^A, e^{A+B} etc. and exponential inner derivations such as $\exp\delta_H$, $\exp(\delta_{H_1}+\delta_{H_2})$ etc. These exponential operators and inner derivations are used very frequently in many-body problems. As it is mostly difficult to diagonalize such exponential operators, it is convenient to find appropriate systematic approximants of them which can be easily evaluated. In Section 2, systematic approximants of e^x are discussed for illustrating our idea. In Section 3, systematic approximants of exponential operators are introduced and studied in detail. Some applications are listed in Section 4.

2. Systematic Approximants of an Exponential Function

In this section we present our idea in a simple exponential function e^x. As is well-known, this is expressed by

$$e^x=\lim_{n\to\infty}(1+x/n)^n\,, \tag{2.1a}$$

or

$$e^x=1+x+x^2/2!+x^3/3!+\ldots+x^m/m!+\ldots\,. \tag{2.1b}$$

The above two formulae give methods to calculate e^x numerically. The second expression (2.1b) is more convenient for such a purpose, because the convergence of (2.1b) is better than that of (2.1a).