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Resistance Inequalities for KMS States of the Isotropic Heisenberg Model*

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Abstract. Inequalities which show that the spin correlations between spins at two lattice sites is bounded by a constant times the inverse square root of the electrical resistance between the lattice sites is proved for KMS states of the isotropic Heisenberg model. The resistance is calculated using the inverse of the coefficients occuring in the Heisenberg Hamiltionian as the resistances between neighboring lattice sites.

Introduction

In this paper we prove inequalities which show that the spin correlation between spins at two lattice sites is bounded by a constant times the inverse square root of the electrical resistance between the lattice sites for KMS states of the isotropic Heisenberg model. The resistance is calculated using the inverse of the coefficients occuring in the Heisenberg Hamiltonian as the resistance between neighboring lattice sites.

The proof of these inequalities comes from combining the ideas of Mermin and Wagner's proof [2] of the absense of ferromagnetism for the isotropic Heisenberg model in one and two dimensions with the resistance type arguments of [3]. A key ingredient of Mermin and Wagner's argument is the use of the Bogoliubov inequality. A short proof of the Bogoliubov inequality for Gibbs states of a full $(n \times n)$-matrix algebra is given beginning on page 130 of Ruelle's book [6]. In the first section of this paper we generalize the Bogoliubov inequality to KMS states of C^*-algebras.

In the second section of this paper we prove resistance inequalities for KMS states of the isotropic Heisenberg model. These inequalities show there is no long range order for the isotropic Heisenberg model in one or two dimensions or for any graph in which the resistance between vertices grows without bound with increasing separation.

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