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Local Cohomology and Superselection Structure

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Abstract. If A^μ is a vector field satisfying $\partial^\mu A^\nu - \partial^\nu A^\mu = 0$ can one find a scalar field ϕ such that $A^\mu = \partial^\mu \phi$? A novel quantum analogue of this classical problem incorporating locality is introduced and is shown to generate those superselection sectors whose charge can be strictly localized. In a 2-dimensional space-time there are further possibilities; in particular, soliton sectors can be generated by this procedure.

1. Introduction

The possibility of describing superselection structure in elementary particle physics in terms of observable quantities was first pointed out by Haag and Kastler [1]. Their idea was that the charge quantum numbers should appear as labels for those inequivalent irreducible representations of the observable algebra whose vector states are relevant to elementary particle physics. The first attempt to give concrete shape to these ideas was undertaken by Borchers [2]. He postulated that the relevant representations were those which are "strongly local equivalent" to the vacuum representation and showed how the unobservable fields could be recovered as the intertwining operators which realize this equivalence. However his postulates, already suspect because his analysis apparently ruled out the possibility of parastatistics, were shown in [3] to be violated if the superselection sectors are generated, as in conventional field theory, by a principle of gauge invariance of the first kind. Nevertheless a slight modification of the terms "strong local equivalence" allowed Borchers' results for systems obeying ordinary Bose and Fermi statistics to be recovered and extended [4].

The systematic treatment of superselection structure in [5, 6] classifies the particle statistics compatible with locality and analyses the operations of charge addition and charge conjugation. On its own terms, this analysis is rather complete; it is true that it has not yet proved possible to show that the superselection structure may always be described as the representation theory of some compact gauge group. However, even if such a result holds, it would add little to our