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Existence of the Critical Point in ϕ^4 Field Theory*

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Abstract. We consider the ϕ^4 quantum field theory in two and three spacetime dimensions. In the single phase region the physical mass (inverse correlation length) $m(\sigma)$ decreases continuously to zero as the bare mass parameter σ approaches a critical value σ_c from above. In three dimensions the critical point σ_c is in the single phase region and the physical mass vanishes there, $m(\sigma_c)=0$.

A consequence of our results is that the critical exponent ν governing the approach to infinite correlations is bounded below (rigorously) by its classical value, 1/2.

I. Introduction and Results

In this paper we show that in the single phase region, the physical mass of the $\lambda{:}\phi^4{:}_d+\sigma{:}\phi^2{:}_d$ quantum field theory, for space-time dimension $d=2,3$, is a continuous increasing function of σ which assumes all strictly positive values. From the point of view of physics this is important since it ensures that by a suitable choice of coupling constants these theories can describe particles of any assigned mass; in short, the theory is mass renormalizable.

Let $\langle\ \rangle_\sigma$ denote expectations for the $\lambda{:}\phi^4{:}_d+\sigma{:}\phi^2{:}_d$ euclidean quantum field theory, obtained as a limit of expectations $\langle\ \rangle_{\sigma,L}$ for the half-Dirichlet theory in volume L, see [1, 2] for details. We fix the Wick ordering mass μ_0 throughout the paper. The long range order $\mathscr{L}(\sigma)$ and the energy gap $\mu(\sigma)$ are defined by:

$$\begin{aligned}\mathscr{L}(\sigma)^2 &= \lim_{|r|\to\infty}\langle\phi(0)\phi(r)\rangle_\sigma\,,\\ \mu(\sigma) &= -\lim_{|r|\to\infty}|r|^{-1}\ln\langle\phi(0)\phi(r)\rangle_\sigma\,.\end{aligned}\tag{1.1}$$

The set $\Sigma\equiv\{\sigma|\mathscr{L}(\sigma)=0\}$ of zero long range order is the single phase region where these models are known to have a unique vacuum, see Simon [2]. By the *GKS* inequalities [2, 3, 4], $\mathscr{L}(\sigma)$ is decreasing in σ. Thus Σ is a proper right half-

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