

Dilations of Dynamical Semi-Groups

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Abstract. We prove the existence of isometric and unitary dilations of a class of semi-groups of completely positive maps on an algebra of operators on a Hilbert space. The result has relevance to the problem of embedding an open quantum mechanical system in a closed one.

§ 1. Introduction

Empirical semi-group laws for the irreversible evolution of the state of a quantum mechanical system have been remarkably successful in a variety of applications [1, 2, 8, 14]. This has encouraged some workers to propose axioms for dynamical semi-groups [10, 12, 7]. From the point of view of fundamental theory such semi-groups are by themselves unsatisfactory: the conventional position is that the laws of quantum theory prescribe the time-reversible evolution of a closed system, and irreversible behaviour enters only when the evolution is restricted to an open sub-system. The time-reversible evolution of a closed system is described by a strongly-continuous one-parameter group of unitary operators on a Hilbert space. The question then arises: is a given irreversible dynamical semi-group the restriction to an open subsystem of a time-reversible evolution of a closed system? The purpose of this paper is to formulate this question mathematically and to answer it in the affirmative for a class of dynamical semi-groups which have interesting applications.

From the mathematical point of view we prove results for semi-groups of completely positive normal maps of W^* -algebras which are analogues of Szökefalvi-Nagy's dilation theorem [17] for semi-groups of contractions on Hilbert spaces and Stroescu's dilation theorem [16] for semi-groups of contractions on Banach spaces. Some results in this direction were obtained by Davies [5]; his proof was based on his theory [4] of quantum jump processes. We adopt his construction of a semi-group of isometries but our proof uses only the perturbation theory of semi-groups on a Banach space.