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Random Point Processes and DLR Equations

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Abstract. We prove the uniqueness of a solution of the Dobrushin-Lanford-Ruelle equation for random point processes when the generating function (interaction potential) has no hard cores, is non-negative and rapidely decreasing.

Introduction

One of the interesting problems in the theory of random processes (r.p.) and random fields (r.f.) which is inspired by mathematical physics (more precisely, by the theory of phase transitions in systems with infinitely many degrees of freedom) is that of their description. The first formulation and motivation of this problem was given by Dobrushin who considered the problem of finding r.f. with a given system of the conditional probabilities [1–2], proved some sufficient conditions of the existence and uniqueness of such r.f. [1–6] and gave examples of non-uniqueness [4].

An independent approach was proposed by Lanford and Ruelle [7–8]. The Lanford-Ruelle approach is more immediate from the point of view of statistical mechanics: given a potential describing the "interaction" of single values of r.f., how many r.f. correspond to it? The uniqueness of the solution of this problem is associated with the absence of the phase transitions in the infinite system with this "interaction".

On the other hand, Dobrushin's approach was developped in [9–11] where the problem of describing the system of conditional probabilities was considered. It turned out that under same general conditions a system of conditional probabilities admits a so-called Gibbs description in terms of a function (or a family of functions) interpreted as a "generalized potential" of interaction of single values of a r.f. We shall call this function the "generating function" (g.f.) of the r.f. The results of [9–11] unite both approaches mentioned above. One of the

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