Commun. math. Phys. 50, 79—85 (1976)

Communications in
Mathematical
Physics

Infrared Bounds, Phase Transitions and Continuous Symmetry Breaking*

J. Fröhlich and B. Simon**
Department of Mathematics, Princeton University, Princeton, New Jersey 08540, USA

T. Spencer
Department of Mathematics, The Rockefeller University, New York, New York 10021, USA

Abstract. We present a new method for rigorously proving the existence of phase transitions. In particular, we prove that phase transitions occur in $(\phi \cdot \phi)_3^2$ quantum field theories and classical, isotropic Heisenberg models in 3 or more dimensions. The central element of the proof is that for fixed ferromagnetic nearest neighbor coupling, the absolutely continuous part of the two point function in k space is bounded by $0(k^{-2})$. When applicable, our results can be fairly accurate numerically. For example, our lower bounds on the critical temperature in the three dimensional Ising (resp. classical Heisenberg) model agrees with that obtained by high temperature expansions to within 14% (resp. a factor of 9%).

§ 1. Introduction

In this paper we develop a new method for establishing the existence of phase transitions or symmetry breaking for a class of ferromagnetic systems in $\nu \geqq 3$ dimensions. In particular we establish symmetry breaking for the classical isotropic Heisenberg model and the $(\phi \cdot \phi)_3^2$ quantum field model. We also establish phase transitions for a broad class of lattice models having no internal symmetry. Work on this last issue has been done by Pirogov and Sinai [38] by very different methods.

It is well known [34, 35, 25, 3, 6, 2] that the $(\phi \cdot \phi)^2$ model and, for finite range interactions, the isotropic Heisenberg model do not exhibit symmetry breaking in one or two dimensions. However for the case of long range interactions or in the presence of anisotropy phase transitions do occur in 2 or more dimensions [15, 40, 1A, 33, 29].

To describe our strategy, let us consider the method of proof for the classical ν-dimensional simple cubic Heisenberg model [34, 26]. Let $F(\alpha-\beta)$ be the two point function $\langle \sigma_\alpha \cdot \sigma_\beta \rangle$ in the infinite volume periodic classical Heisenberg model

* Research supported by USNSF under grants GP-38048 and MPS-74-13252
** A. Sloan Fellow; also in the Department of Physics