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Integration of the Tomonaga-Schwinger Equation

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Abstract. Some integrations of the Tomonaga-Schwinger equation with a non-local interaction are studied with mathematical rigor. It is proved that the related initial value problem has a unique solution in any finite region of the space-time corresponding to each set of space-like surfaces which covers the region. Such an analysis can be extended to the case of quantum electrodynamics by the aid of a Lorentz-invariant topology introduced in the *-algebra of electromagnetic field operators.

§ 1. Introduction

Though the Tomonaga-Schwinger equation

$$i\delta\Psi(\sigma)/\delta\sigma(x)=H(x)\Psi(\sigma) \tag{1.1}$$

in the interaction picture is most fundamental in the early quantum field theory, it is very difficult to understand the precise mathematical meaning of this equation. If, however, we replace the interaction Hamiltonian density $\underset{\sim}{H}(x)$ by some non-local ones, we get a clue to treat the equation mathematically. The purpose of the present paper is to study this problem in some detail.

We start with the scalar coupling between a neutral scalar field of mass $\mu \neq 0$ and a spinor field of mass $\kappa \neq 0$. We work with the Hilbert space

$$\mathfrak{H}=\mathfrak{H}_M \otimes \mathfrak{H}_D ,$$

where $\mathfrak{H}_M$ is the Fock space for creation and annihilation operator-valued distributions $\phi^*(\boldsymbol{k})$ and $\phi(\boldsymbol{k})$ of the neutral scalar field and $\mathfrak{H}_D$ is the same for the spinor field. We are interested in the local interaction Hamiltonian density

$$H(x)=gN[\tilde{\psi}(x)\psi(x)]\phi(x)^{1} , \tag{1.2}$$

[1] $N[\ldots]$ means the normal product