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Constants of Motion in Local Field Theory (Coleman's Theorem Revisited)

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Abstract. We analyze the space integrals $Q = \int d^3x\varrho(\mathbf{x})$ of finitely localized densities ϱ. It turns out that the time translated operators $Q(t)$ are polynomials in t if Q annihilates the vacuum. In particular, $Q(t) = Q$ in models with short-range forces and complete particle interpretation. These results are valid in the Haag-Araki framework of field theory as well as in the Wightman formalism. Lorentz covariance is not needed in the proofs.

Introduction and Main Results

In a recent paper Gal-Ezer and Reeh have shown that the space-integral of the zeroth component of a tensor current defines a conserved charge if it annihilates the vacuum [1]. This work generalizes a result which is known in the literature as Coleman's theorem [2] and it adds to a series of papers which were stimulated by Coleman's original article. (See for example [3] and the references therein.) Although there may be differences in style and rigour the idea of proof in all these investigations is essentially the same. The argument is based on a detailed study of the two-point function of the divergence of the current. It follows from Lorentz-covariance that the intermediate states which contribute to this function have zero mass if the charge annihilates the vacuum. Thus the two-point function vanishes in theories with a mass gap and the current is conserved. In the presence of massless particles one cannot conclude quite as much. There the recent result of Gal-Ezer and Reeh is the best one to be expected.

Since Lorentz-covariance is very essential for the above argument one may ask whether Coleman's theorem depends crucially on this assumption and it is the aim of the present paper to clarify this point. We shall show that locality and spectrum condition are already sufficient ingredients for a proof and that Lorentz-covariance is not needed. Moreover, in an appropriate formulation Coleman's theorem holds also for finitely localized quantities like Haag-Araki fields or locally smeared polynomials in the basic Wightman-fields. It is in the latter case in which we can formulate the most easily comprehensible version of our main