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# On the Space-Time Interpretation of Classical Canonical Systems
# II: Relativistic Canonical Systems

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**Abstract.** Relativistic canonical systems and their symmetries are defined and classified within the class of canonical systems treated in a previous paper. Their algebra of variables contains a subset of "monotone" variables which satisfy a certain uniqueness condition and are later shown to increase strictly in the course of the dynamical evolution of the system on all physically acceptable states. This leads to a unique space-time interpretation of relativistic canonical systems. Finally we study space-time factorizations of such systems and introduce the appropriate notion of states. For a certain simple class of states the theory is shown to describe the motion of relativistic matter in some external gravitational and electromagnetic field.

## 1. Introduction

In this paper we shall study a certain class of canonical systems, the general theory of which we have developed in [1]. Let us briefly state the basic notions and results obtained there.

A canonical system is an ordered set containing a canonical manifold $M$ (with canonical form $\Omega$, see [2]) and a canonical vectorfield $Y$ on $M$ (the kinematical vectorfield). The algebra $\mathfrak{A}(M)$ of differentiable functions on $M$ (these are called variables) contains a subalgebra $\mathfrak{A}_0$ which is required to satisfy a set of Kinematical Axioms: Under the Poisson bracket operation $\mathfrak{A}_0$ is maximal commutative and is mapped to itself by variables from the subset $Y(\mathfrak{A}_0)$. Both $\mathfrak{A}_0$ and $Y(\mathfrak{A}_0)$ have only the zero variable in common and determine the differentiable structure on $M$ (such sets of functions which define a differentiable structure on $M$ are called sufficient sets).

The Hamiltonian vectorfields generated by variables in $\mathfrak{A}_0$ define a quotient manifold $N$ of integral submanifolds in $M$. To any variable $A$ in $\mathfrak{A}_0$ there corresponds a unique differentiable function $A^*$ on $N$, and vice versa. The vectorfields $X$ on $N$ are in bijective correspondence with variables $P_X$ in some submodule of functions $\mathfrak{A}_1$:

$$\{P_X, A\}^* = X(A^*);\ A \text{ in } \mathfrak{A}_0,\ P_X \text{ in } \mathfrak{A}_1\,. \tag{1.1}$$