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On the Space-Time Interpretation of Classical Canonical Systems
I: The General Theory

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Abstract. We define a canonical system as a canonical manifold M plus a canonical vectorfield on M. For such systems a unique kinematical interpretation is deduced from a set of Kinematical Axioms satisfied by the algebra of differentiable functions on M. This algebra is required to contain a subalgebra which is maximal commutative under the Poisson bracket. M is shown to be diffeomorphic to the cotangent bundle over its quotient manifold, which is defined by the given subalgebra. Canonical systems satisfying these axioms are then classified. If the "phase space interpretation" is adopted they are shown to describe the motion of masspoints in some configuration space under the influence of and interacting by arbitrary vector and scalar potentials.

1. Introduction

In this paper we study the space-time interpretation of classical canonical systems. At a first glance this would seem to pose no problem. After all classical physics is concerned with phenomena occuring in space and time such as the motion of material bodies and their relation to the classical fields. Thus the basic theoretical quantities and dynamical principles are formulated in terms of space-time concepts, and no additional interpretation is required.

However a different situation is met if we wish to consider a classical theory as the limiting case of some corresponding quantum theory. In this case the space-time interpretation of the classical theory should itself be obtainable from some elements of structure which are already present in the quantum theory. Let us consider the algebraic formulation of quantum theory [1]. In this formulation operations corresponding to measurements are represented by elements in some non-commutative $*$-algebra over the complex numbers. The possible states of the physical systems under consideration are represented by positive linear functionals on the given algebra, which assign real numbers as "expectation values" to those elements which represent measurements. The