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Hyperfunction Quantum Field Theory II. Euclidean Green's Functions

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Abstract. The axioms for Euclidean Green's functions are extended to hyperfunction fields without being supplemented by any condition like the linear growth condition of Osterwalder and Schrader.

§ 1. Introduction

In a previous paper [1], which will be quoted as NM I, we have formulated the quantum field theory in terms of Fourier hyperfunctions, successfully in showing that Wightman's axioms for tempered fields can be extended to hyperfunction fields. In particular we have manifested that the support concept of Fourier hyperfunctions allows us to state the locality axiom, in spite of the disadvantage that the test function space for hyperfunctions contains no C^∞ functions of compact support.

It is quite natural that questions may arise whether the hyperfunction fields work effectively in the scattering theory, dispersion relations and other provinces of the quantum field theory relating to local singularity structure or momentum space analyticity properties of fields. Another interesting question perhaps concerns the Euclidean formulation of the hyperfunction quantum field theory. In order to get a reconstruction theorem for tempered fields satisfying the usual Wightman axioms, Osterwalder and Schrader [2, 3] were compelled to introduce a technical axiom, what they called the linear growth condition, in formulating the axioms for Euclidean Green's functions. When tempered fields are replaced by hyperfunction fields, which are of a class wider than the former, can the technical axiom such as the linear growth condition be removed completely? In other words, one may ask whether we can formulate a set of axioms for Euclidean Green's functions which contains *neither* the linear growth condition *nor* something else and is *equivalent* to a set of axioms for Wightman hyperfunctions set up in NM I.

The last question is answered affirmatively in the present and a subsequent papers. To this end, however, we find it necessary to make a slight modification