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Correlation Inequalities for Multicomponent Rotators

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Abstract. A recent approach to G.H.S. and Lebowitz inequalities is used to prove Griffiths' second inequality for 3 and 4 component models (e.g. Classical Heisenberg model, $|\varphi|^4$ Euclidean fields). Applications include monotonicity of the mass gap in the external field, and two-sided inequalities between "parallel" and "transverse" correlations.

1. Introduction

The inequalities proven by Ginibre for ferromagnetic plane rotators are very powerful [1, 2]. Their proof, however, depends essentially on the commutative structure of the circle group, through the use of characters. Elementary trigonometry and a very special symmetry were also used in [1], but we show in Section 4 that these can be avoided if one is only interested in positive correlations of vectors, e.g.

$$\langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle \geqq \langle \boldsymbol{S}_i \rangle \cdot \langle \boldsymbol{S}_j \rangle . \tag{1.1}$$

Namely, for D-dimensional classical ferromagnets, we reduce (1.1) to a *first* Griffiths inequality for a similar system with interactions of the form

$$(\hat{x}, U(i)^{-1} U(j)\hat{x}) + (\hat{y}, U(i)^{-1} U(j)\hat{y}) \tag{1.2}$$

where $\hat{x}$ and $\hat{y}$ are fixed orthogonal unit vectors in $\mathbb{R}^D$ and $U(j)$, for each site j, is to be integrated over the rotation group acting in $\mathbb{R}^D$ with the Haar measure. In the commutative case ($D=2$), the functions (1.2) are positive definite on the product group (over the sites), which is more than what we need. As a by-product we prove that the parallel mass gap is larger than the transverse mass gap (see 1.3).

We do not know how to deal with non commutative rotations. This article is mainly devoted to 3 and 4 component models, where each spin may be considered as a plane rotator plus an Ising (continuous) spin, or as two plane rotators, negatively correlated. Of course we'll never have in this way more than two components on the same footing in the correlations, and therefore no scalar product, but all the