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Time Evolution for Infinitely Many Hard Spheres

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Abstract. We construct the time evolution for infinitely many particles in $\mathbb{R}^\nu$ interacting by the hard-sphere potential

$$\Phi(x)=\begin{cases}+\infty & |x|<a\\ 0 & |x|\geqq a\,.\end{cases}$$

Because there are abundant examples of hard-sphere configurations with more than one solution to the Newtonian equations of motion, we introduce the concept of a *regular* solution, in which the growth of velocities and crowding of particles at infinity are limited. We prove that (1) regular solutions exist with probability one in every equilibrium state, and (2) any configuration of the infinite system is the initial point of at most one regular solution. Equilibrium states are invariant under the time-evolution.

0. Introduction

0.1. Imagine infinitely many billiard balls of mass m and diameter a at positions $q_i \in \mathbb{R}^\nu$, $i=1,2,\dots$ (where $|q_i-q_j|\geqq a$ if $i \neq j$), with corresponding momenta $p_i \in \mathbb{R}^\nu$, $i=1,2,\dots$. The problem is to solve the Newtonian equations of motion when these particles interact by the hard-sphere potential

$$\Phi(x)=\begin{cases}+\infty & |x|<a\\ 0 & |x|\geqq a\,.\end{cases} \tag{0.1.1}$$

The equations of motion take, roughly speaking, the form

$$\begin{aligned}\dot{q}_i &= p_i/m\\ \dot{p}_i &= 0 + \text{elastic reflection at collisions}\,.\end{aligned} \tag{0.1.2}$$

Rigorous work on time-evolution of classical systems of infinitely many particles was pioneered by Lanford [4–5]. Lanford's 1974 Battelle lectures [6] contain

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