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The Geometry of a Covariant Expansion in Chiral Theories with Nucleons

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Abstract. We deal with the geometrical foundations of a covariant scheme, which is developed to give a "co-ordinate independent" perturbation expansion of $SU(2)\times SU(2)$ chiral theories with pions and nucleons. The pion fields play the role of local co-ordinates on a 3-D manifold with constant curvature in isospace. The presence of the nucleon isospinor field forces us to deal with the problem of endowing the manifold with a spin structure. In this way the nucleon isospinor is accommodated in the fiber space of the principal fiber bundle of the tangent bundle of the manifold.

0. Introduction

The non-linear realizations of the chiral group $G=SU(2)\times SU(2)$ are studied from the geometric point of view in [1]. In this reference the 3-dimensional non-linear realization associated with the pion isovector field is considered as a group of co-ordinate transformations on a 3-dimensional isospace of constant curvature leaving invariant the line element. In this treatment the pion field components are taken to be the co-ordinates in the curved isospace. The various non-linear models (i.e. chiral invariant Lagrangian densities which are functions of the pion field only) result from a specific choice of the co-ordinate system used to parametrize the manifold. It is known, however, that the S-matrix elements should be independent of the choice of the pion field in term of which the Lagrangian density is defined. A co-ordinate independent formulation of perturbation theory is developed in [2, 3]. In the present article we would like to extend the formalism of [2, 3] to cover the case of $SU(2)\times SU(2)$ invariant Lagrangian densities which are functions of the pion isovector as well as the nucleon isospinor fields. Consequently we are forced to deal with the problem of "laying down a spinor field" upon the manifold under consideration, that is, it is necessary to endow the manifold with a spin structure. This structure is defined by means of a field of driads. The dreibein fields, therefore, play an essential role in our formalism.