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Space-Time Singularities

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Abstract. A set of conditions for the reasonableness of space-time is proposed and investigated. Using these, together with strong causality and an assumption of genericness, it is shown that future timelike or null geodesically incomplete space-times contain either curvature or intermediate singularities, or primordial singularities.

1. What is a Reasonable Space-Time?

One would like to find acceptable physical grounds for excluding many of the "pathological" spacetimes that can be constructed as counter-examples to seemingly plausible conjectures. For instance, it might be thought that gravitational collapse would inevitably lead to a curvature or intermediate singularity [1]; it would, however, be mathematically possible for space-time simply to come to an end before any predicted singularity formed. To prevent this, I shall propose two physical conditions that space-time should satisfy. One (maximality) asserts that space-time does not arbitrarily stop; the other (hole-freeness) asserts that predictions, and perhaps retrodictions, made on the basis of formally adequate Cauchy data are not falsified by the spontaneous appearance of uncaused singularities.

A further condition, rather weaker than the Hausdorff conditions, requires that a non-quantum space-time (excluding the Wheeler-Everett picture) does not undergo arbitrary branching. This leads to the concept of a Hajicek space-time [2, 3].

In what follows "smooth" denotes some fixed sufficiently strong differentiability condition on the metric. "Singularity" is used in the sense of Schmidt [7].

Definition 1. *A Hajicek space-time (or simply: a space-time) is a pair* (M, g)*; where* M *is a connected* C^∞ *4-manifold, not necessarily Hausdorff,* g *is a smooth pseudo-Riemannian metric on* M *of signature* $(- + + +)$*, and* M *has the Hajicek property: there exists no pair of curves* $c_i:(0, 1]\to M$ $(i=1, 2)$ *for which* $c_1|(0, g)=c_2|(0, g)$ *but* $c_1(g)\neq c_2(g)$ *for some* $g\in(0, 1]$.