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Zero-Mass Limit in Perturbative Quantum Field Theory

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Abstract. A new R-operation which satisfies Bogolubov-Parasiuk and Hepp recurrence and which is infrared and ultra violet convergent graph by graph, is defined in perturbative quantum field theory. This new subtraction scheme is used to achieve the zero-mass limit of a massive field theory.

I. Introduction

In 1970 Callan [1] and Symanzik [2] introduced a differential equation to study the high energy behaviour of renormalized vertex functions in Lagrangian field theory. By homogeneity arguments, the asymptotic high energy behaviour of these functions also describes their infrared limits when all masses m_i in the theory tend to zero. For strictly renormalized field theory, it is found that such infrared limits exist at nonexceptional momenta provided that the divergent vertex functions are subtracted in a convenient fashion [3]. This result was also proved by Gell-Mann and Low using the technique of the renormalization group [4].

This paper intend to show the detailed mechanism of the infrared limit. In a recent publication [5] we have obtained explicitely the asymptotic high energy behaviour of a renormalized Feynman amplitude subtracted at zero momentum; we have especially given specific "geometric" rules to construct the coefficients of all powers of logarithm for the leading power behavior. These rules are given at the end of this introduction for the case of graphs generated by a strictly or non renormalizable field theory at non-exceptional momenta. In that case, the structure of these coefficients is such, that there exists for any graph a linear combination of Feynman amplitudes which has a zero-mass limit. Namely, the powers of logarithm which arise in the zero-mass limit of each Feynman amplitude when subtracted at zero momentum, are cancelled by the logarithms which enter the coefficients of the linear combination. It is the purpose of Section II to show this

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