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Applications of the Stochastic Ising Model to the Gibbs States*

Richard A. Holley and Daniel W. Stroock

Department of Mathematics, University of Colorado, Boulder, Colorado, USA

Abstract. The stochastic Ising model is used as a tool to prove theorems concerning analyticity of the correlation functions and strong cluster properties of the Gibbs states.

0. Introduction

The stochastic Ising model has been used as a model for the time evolution of the configuration of spins in the classical Ising model. From a physical point of view the model has the unfortunate feature that the dynamics do not come from a Hamiltonian and are not well motivated. Nevertheless it is possible to learn something about a Gibbs state by studying the semi-group of the stochastic Ising model which has that Gibbs state as its stationary measure. The results proved in this paper demonstrate this technique.

Let Z^d be the d-dimensional integer lattice and let $\{J_R : R \text{ a finite subset of } Z^d\}$ be a potential which satisfies

(0.1) $J_R = J_{R+k}$ for all $R \subset Z^d$ and $k \in Z^d$

and

(0.2) $\sum_{R \ni 0} |J_R| < \infty$.

Let $E = \{-1, 1\}^{Z^d}$ be the set of configurations of spins and give E the product topology. The elements of E are denoted by letters such as η or σ, and we denote the spin at k in the configuration η by η_k. Let $\mathscr{B}$ be the Borel sets of E and if $F \subset Z^d$ let $\mathscr{B}_F(\tilde{\mathscr{B}}^F)$ denote the σ-algebra generated by $\{\eta_k : k \in F\}$ $(\{\eta_k : k \notin F\})$. We say a probability measure μ on $\mathscr{B}$ is a Gibbs state for the potential $\{J_R\}$ if a regular conditional probability distribution of μ on $\mathscr{B}_{\{k\}}$ given $\tilde{\mathscr{B}}^{\{k\}}$ is given by

(0.3) $$\varrho_k(\{\eta_k\}|\tilde{\eta}^k) = \left[1 + \exp\left[2 \sum_{R \ni k} J_R \prod_{j \in R} \eta_j\right]\right]^{-1}.$$

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