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Self-Adjointness and Invariance of the Essential Spectrum for Dirac Operators Defined as Quadratic Forms

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Abstract. Some general results about perturbations of not-semibounded self-adjoint operators by quadratic forms are obtained. These are applied to obtain the distinguished self-adjoint extension for Dirac operators with singular potentials (including potentials dominated by the Coulomb potential with $Z < 137$). The distinguished self-adjoint extension, is the *unique* self-adjoint extension, for which the wave functions in its domain possess finite mean kinetic energy. It is shown moreover that the essential spectrum of the distinguished extension is contained in the spectrum of the free Hamiltonian.

1. Introduction

In this paper we shall consider the problems of self-adjointness and of the invariance of the essential spectrum for the Dirac operator perturbed by a local potential. The formal Hamiltonian to be considered is

$$-i\alpha\cdot\text{grad}+m\beta+V(x)\,. \tag{1.1}$$

Reviews concerning the self-adjointness problem both in relativistic and non-relativistic quantum mechanics appeared recently. See [2] where an extensive bibliography is also given, and [3] which considers also nonlocal perturbations. The status of the theory is almost satisfactory due to the results obtained in the last years. However, there is a point in which the relativistic theory is less satisfactory: the case of singular potentials when the minimal operator (1.1) is not essentially self-adjoint. In the nonrelativistic (Schrödinger) case the semiboundedness of the sesquilinear form defined by the minimal operator is sufficient to provide a distinguished self-adjoint operator in a canonical way, the Friedrichs extension, which is taken to represent the physical Hamiltonian irrespective of the fact that the minimal operator is essentially self-adjoint or not. In the relativistic case (due to the unboundedness of the free Dirac operator) a general method to