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Operator Product Expansions in Conformally Covariant Quantum Field Theory

W. Rühl and B. C. Yunn

Fachbereich Physik der Universität, D-6750 Kaiserslautern, Federal Republic of Germany

Abstract. Operator products in quantum field theory on two-dimensional Minkowski space are expanded into a series of local operators by means of the tensor product decomposition theorem for representations of the conformal group. The Thirring model is used as an explicit example. Two types of expansions result. If the operator product acts on the vacuum state, we obtain strictly covariant expansions. In general however, each term in the expansion is only semicovariant.

1. Introduction

Operator product expansions in conformal invariant quantum field theory have been studied [1–4] as a means to get insight into operator product expansions of more general and physically more relevant quantum field theories. In fact the requirement of conformal invariance restricts the structure of quantum field theory very strongly. This supports the hope that some problems of quantum field theory may become solvable by a nonperturbative construction in such models.

The general treatment of operator product expansions ("Wilson expansions" [5]) has been to multiply two local operators $A(x)$ and $B(y)$, guess an infinite set of other local operators $C_n(z)$, make an ansatz for the expansion of $A(x)B(y)$ in terms of the $C_n(z)$ in the sense of an asymptotic expansion for $x \to y$, and impose the requirement of conformal invariance on this expansion at the end.

Our approach is different. We intend to apply the reduction theorem of tensor products of representations for the conformal group into irreducible representations to this problem. Thus we want to find all terms in the expansion by construction. The idea is to guarantee "completeness" of the operators $C_n(z)$ this way. Of course this construction can only be matrix-element-wise and representation by representation (each local operator belongs to an infinite number of representations corresponding to different charge sectors). At the end there remains the problem to identify these matrix elements as elements of a known local field operator.